Question posée : Calculer $\sum_{k=0}^{k=n} \cos(kx)$.

En utilisant une formule classique

$$\begin{aligned}\sin a \cos b &= \frac{1}{2}(\sin(a+b) + \sin(a-b)) \\ &= \frac{1}{2}(\sin(b+a) - \sin(b-a))\end{aligned}$$

Avec kx et $\frac{x}{2}$ à la place de b et de a on obtient

$$\begin{aligned}
\sum_{k=0}^{k=n} \cos(kx)\sin(\frac{x}{2}) &= \sum_{k=0}^{k=n} \frac{1}{2}(\sin(kx+\frac{x}{2}) - \sin(kx-\frac{x}{2})) \\
\sum_{k=0}^{k=n} \cos(kx) &= \frac{1}{2\sin(\frac{x}{2})} \sum_{k=0}^{k=n} \sin(kx+\frac{x}{2}) - \sin(kx-\frac{x}{2}) \\
\sum_{k=0}^{k=n} \cos(kx) &= \frac{1}{2\sin(\frac{x}{2})} \left(\sum_{k=0}^{k=n} \sin(kx+\frac{x}{2}) - \sum_{k=0}^{k=n} \sin(kx-\frac{x}{2}) \right) \\
&= \frac{1}{2\sin(\frac{x}{2})} \left(\sum_{k=0}^{k=n} \sin(kx+\frac{x}{2}) - \sum_{k=-1}^{k=n-1} \sin(kx+\frac{x}{2}) \right) \\
&= \frac{1}{2\sin\frac{x}{2}} \left(\sum_{k=0}^{k=n-1} \sin(kx+\frac{x}{2}) + \sin(nx+\frac{x}{2}) - \sum_{k=0}^{k=n-1} \sin(kx+\frac{x}{2}) - \sin(-\frac{x}{2}) \right) \\
&= \frac{1}{2\sin(\frac{x}{2})} \left(\sin(nx+\frac{x}{2}) - \sin(-\frac{x}{2}) \right) \\
&= \frac{1}{2\sin(\frac{x}{2})} \left(\sin(nx+\frac{x}{2}) + \sin\frac{x}{2} \right) \\
\sum_{k=0}^{k=n} \cos(kx) &= \boxed{\frac{\sin(nx+\frac{x}{2}) + \sin\frac{x}{2}}{2\sin(\frac{x}{2})}} \\
&= \frac{2\sin\frac{nx+x}{2}\cos\frac{nx}{2}}{2\sin(\frac{x}{2})} \\
&= \frac{\sin\frac{nx+x}{2}\cos\frac{nx}{2}}{\sin(\frac{x}{2})} \\
\sum_{k=0}^{k=n} \cos(kx) &= \boxed{\frac{\sin\frac{(n+1)x}{2}\cos\frac{nx}{2}}{\sin(\frac{x}{2})}}
\end{aligned}$$